Workshop

Analysis of transport equations:

Vlasov and related models

Talk given by

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Title. *Continuous and discrete variational principles for the Vlasov-Maxwell and related gyrokinetic models.*

Abstract. The Vlasov-Maxwell equations can be derived from a variational principle using either a Eulerian formulation of the Vlasov equation, where the distribution function f is varied, along with the potentials ? and A for the fields, or a Lagrangian formulation of the Vlasov equation, where the characteristics X and V are varied instead of f. We are going to consider the latter version, which was introduced by Low in 1958. It has been realized around 2000 by Sugama for the Lagrangian case and Brizard for the Eulerian case, that the self-consistent gyrokinetic approximation of the Vlasov-Maxwell equations could be conveniently derived by just replacing the single particle Lagrangian by a relevant approximation in the variational principle. This is one of the pillars of modern gyrokinetic theory. We are going to show how different types of gyrokinetic models can be obtained within this framework and how this leads to exact conservation of a modified energy and angular momentum. In the second part of the talk we are going to show how plugging a particle approximation of the distribution function and a compatible Finite Element approximation of the potentials naturally leads to a semi-discrete Poisson structure, time remaining continuous. This leads to an exact conservation of some semi-discrete invariants: the energy and two Casimir related to div B and the Gauss law. Possible time discretizations of the semi-discrete equations exactly conserving some of these invariants will also be discussed.